Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Monday 16 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43159A

1. A particle A has constant velocity $(3\mathbf{i} + \mathbf{j})$ m s^{-1} and a particle B has constant velocity $(\mathbf{i} - \mathbf{k})$ m s^{-1}. At time $t = 0$ seconds, the position vectors of the particles A and B with respect to a fixed origin O are $(-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$ m and $(-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ m respectively.

(*a*) Show that, in the subsequent motion, the minimum distance between A and B is $4\sqrt{2}$ m.
(6)

(*b*) Find the position vector of A at the instant when the distance between A and B is a minimum.
(2)

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2. A car of mass 1000 kg is moving along a straight horizontal road. The engine of the car is working at a constant rate of 25 kW. When the speed of the car is v m s^{-1}, the resistance to motion has magnitude $10v$ newtons.

(*a*) Show that, at the instant when $v = 20$, the acceleration of the car is 1.05 m s^{-2}.
(3)

(*b*) Find the distance travelled by the car as it accelerates from a speed of 10 m s^{-1} to a speed of 20 m s^{-1}.
(8)

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3. A small ball is moving on a smooth horizontal plane when it collides obliquely with a smooth plane vertical wall. The coefficient of restitution between the ball and the wall is $\frac{1}{3}$. The speed of the ball immediately after the collision is half the speed of the ball immediately before the collision.

Find the angle through which the path of the ball is deflected by the collision.
(8)

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4. At noon two ships A and B are 20 km apart with A on a bearing of 230° from B. Ship B is moving at 6 km h^{-1} on a bearing of 015°. The maximum speed of A is 12 km h^{-1}. Ship A sets a course to intercept B as soon as possible.

(*a*) Find the course set by A, giving your answer as a bearing to the nearest degree.
(4)

(*b*) Find the time at which A intercepts B.
(4)

5.

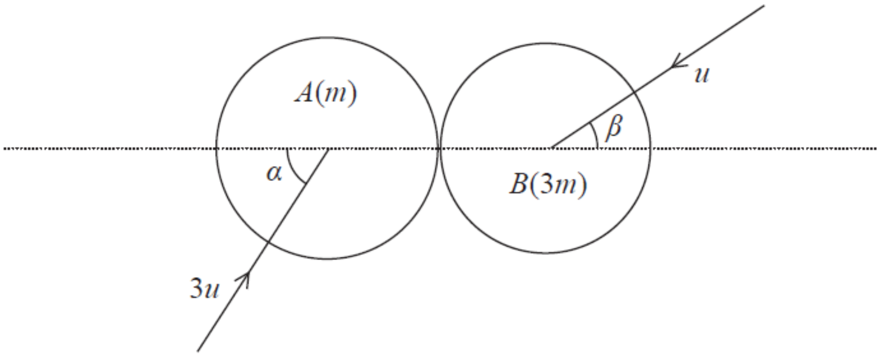

Figure 1

Two smooth uniform spheres A and B have equal radii. The mass of A is m and the mass of B is $3m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before the collision, A is moving with speed $3u$ at angle α to the line of centres and B is moving with speed u at angle β to the line of centres, as shown in Figure 1. The coefficient of restitution between the two spheres is $\frac{1}{5}$. It is given that $\cos\alpha = \frac{1}{3}$ and $\cos\beta = \frac{2}{3}$ and that α and β are both acute angles.

(a) Find the magnitude of the impulse on A due to the collision in terms of m and u.

(8)

(b) Express the kinetic energy lost by A in the collision as a fraction of its initial kinetic energy.

(4)

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6. A particle of mass m kg is attached to one end of a light elastic string of natural length a metres and modulus of elasticity $5ma$ newtons. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle is held at rest on the plane with the string stretched to a length $2a$ metres and then released at time $t = 0$. During the subsequent motion, when the particle is moving with speed v m s^{-1}, the particle experiences a resistance of magnitude $4mv$ newtons. At time t seconds after the particle is released, the length of the string is $(a + x)$ metres, where $0 \leq x \leq a$.

(a) Show that, from $t = 0$ until the string becomes slack,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 0$$

(3)

(b) Hence express x in terms of a and t.

(6)

(c) Find the speed of the particle at the instant when the string first becomes slack, giving your answer in the form ka, where k is a constant to be found correct to 2 significant figures.

(4)

7.

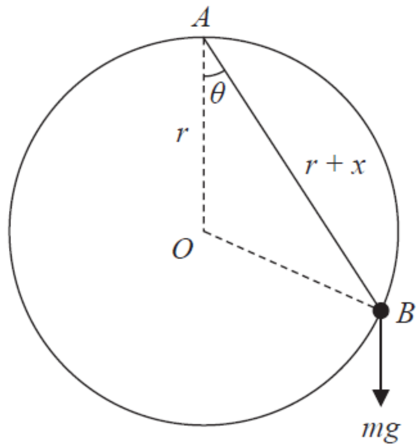


Figure 2

A bead B of mass m is threaded on a smooth circular wire of radius r, which is fixed in a vertical plane. The centre of the circle is O, and the highest point of the circle is A. A light elastic string of natural length r and modulus of elasticity kmg has one end attached to the bead and the other end attached to A. The angle between the string and the downward vertical is θ, and the extension in the string is x, as shown in Figure 2.

Given that the string is taut,

(a) show that the potential energy of the system is

$$2mgr\{(k-1)\cos^2\theta - k\cos\theta\} + \text{constant}$$

(6)

Given also that $k = 3$,

(b) find the positions of equilibrium and determine their stability.

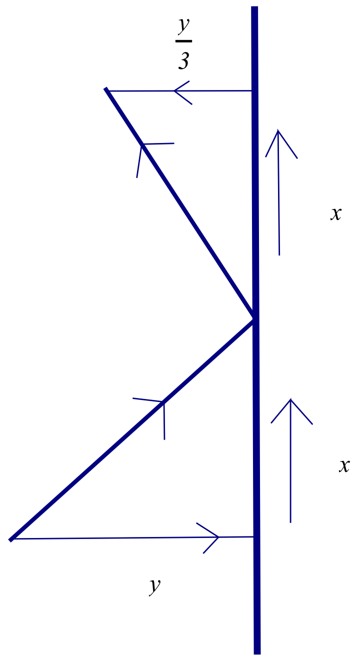
(9)

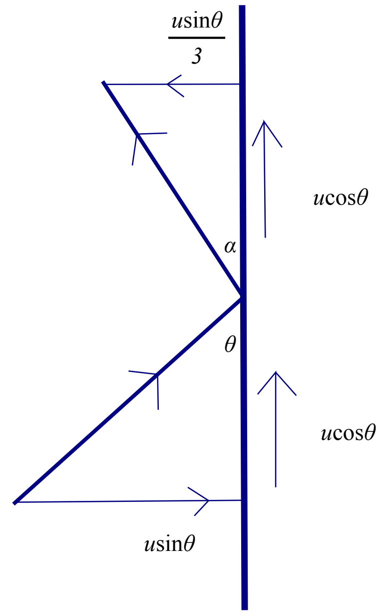
TOTAL FOR PAPER: 75 MARKS

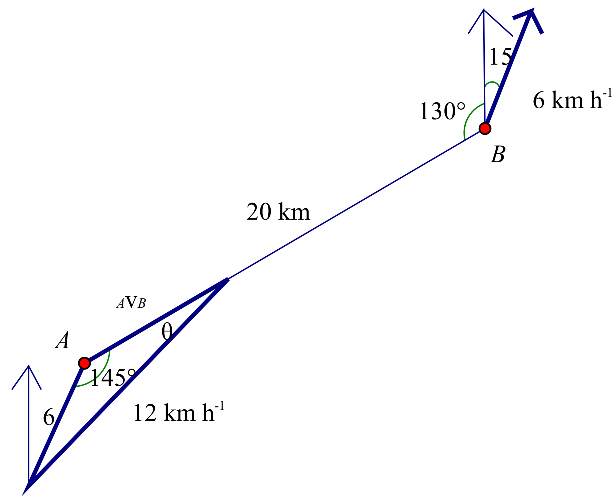
END

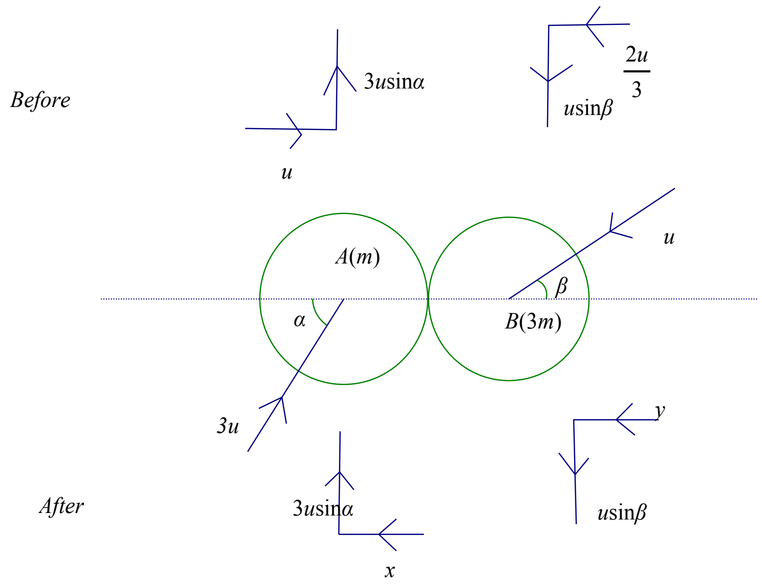
Question Number	Scheme	Marks	Notes
1a	$\mathbf{r}_A = (-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) + t\,(3\mathbf{i} + \mathbf{j}) = ((-6 + 3t)\mathbf{i} + (4 + t)\mathbf{j} + (-3)\mathbf{k})$	M1	Position vector for A or B
	$\mathbf{r}_B = (-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + t(\,\mathbf{i} - \mathbf{k}) = ((-2 + t)\mathbf{i} + (2)\mathbf{j} + (3 - t)\mathbf{k})$	A1	Both position vectors correct (seen or implied)
	${}_B\mathbf{r}_A = (-2 + \mathrm{t} + 6 - 3t)\mathbf{i} + (2 - 4 - t)\mathbf{j} + (3 - t + 3)\mathbf{k}$ $= (4 - 2t)\mathbf{i} + (-2 - t)\mathbf{j} + (6 - t)\mathbf{k}$	M1	Position of B relative to A (or A relative to B)
	$\left\lvert {}_B\mathbf{r}_A \right\rvert^2 = (4-2t)^2 + (t+2)^2 + (6-t)^2$	M1	Use of Pythagoras
alt1	$= 6t^2 - 24t + 56 = 6(t-2)^2 + 32$	M1	Complete the square
	Minimum distance = $\sqrt{32} = 4\sqrt{2}$ m **	A1 [6]	Reach **given answer** correctly
	$\left\lvert {}_B\mathbf{r}_A \right\rvert^2 = (4-2t)^2 + (t+2)^2 + (6-t)^2 \left(= 6t^2 - 24t + 56\right)$	M1	Use of Pythagoras
alt2	$12t - 24 = 0 \Rightarrow t = 2$	M1	Differentiate and solve for t
	Minimum distance = $\sqrt{32} = 4\sqrt{2}$ m **	A1	Reach **given answer** correctly
alt3	$\begin{pmatrix} 4-2t \\ -2-t \\ 6-t \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 \Rightarrow 8 - 4t - 2 - t + 6 - t = 12 - 6t = 0$	M1	Scalar product of position vector with relative velocity = zero and form equation in t
	Distance $= \sqrt{0^2 + 4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$	M1	Use of Pythagoras
		A1	Reach **given answer** correctly
1b	When $t = 2$,	B1	Seen or implied
	$\mathbf{r}_A = 6\mathbf{j} - 3\mathbf{k}$	B1 [2]	cso

Question Number	Scheme	Marks	Notes
2a	$\frac{P}{v}-10v=ma\,;\ \frac{25000}{v}-10v=1000a$	M1	Equation of motion
	$v=20$, (m s^{-2}) $\quad a=\dfrac{\frac{25000}{20}-10\times 20}{1000}=\dfrac{\frac{25}{2}-2}{10}$	DM1	Substitute $v=20$
	$=1.05$ (m s^{-2}) **	A1 [3]	Obtain **given answer** correctly
2b	$v\frac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{\frac{25000}{v}-10v}{1000}=\dfrac{25000-10v^2}{1000v}=\dfrac{2500-v^2}{100v}$	M1 A1	Differential equation in v and x Any equivalent form
	$\int\frac{100v^2}{2500-v^2}\mathrm{d}v=\int 1\mathrm{d}x \qquad \left(=100\int -1+\frac{2500}{2500-v^2}\mathrm{d}v\right)$	M1	Separate the variables
alt1	$=100\int -1+\frac{25}{50-v}+\frac{25}{50+v}dv$	DM1 A1	Split using partial fractions Or equivalent
	$x(+C)=100\left\{-v+25\ln\left\|\frac{50+v}{50-v}\right\|\right\}$	A1	Integration correct
	$x=100\left(-20+25\ln\frac{70}{30}\right)-100\left(-10+25\ln\frac{60}{40}\right)=105\,(\mathrm{m})$	DM1 A1 [8]	Correct use of limits Or better $\left(2500\ln\left(\frac{14}{9}\right)-1000\right)$
alt2	$=100\left(v-50\operatorname{arc\,tanh}\left(\frac{v}{50}\right)\right)$	DM1 A1	Use of arctanh correct
	$x(+C)=100\left\{-v+25\ln\left\|\frac{50+v}{50-v}\right\|\right\}$	A1	Convert to log form
	$x=100\left(-20+25\ln\frac{70}{30}\right)-100\left(-10+25\ln\frac{60}{40}\right)=105\,(\mathrm{m})$	DM1 A1	Correct use of limits Or better $\left(2500\ln\left(\frac{14}{9}\right)-1000\right)$
	NB A correct numerical answer that does not follow from integration scores no marks.		

Question Number	Scheme	Marks	Notes
3 alt1			
	Speed perpendicular to wall after collision = $\frac{y}{3}$	B1	
	Speed parallel to the wall is unchanged	B1	
	$\frac{1}{2}\left(x^2+y^2\right)=x^2+\frac{1}{9}y^2$	M1 A1	Use the speeds to form an equation in x & y (or equivalent) Correct unsimplified
	$9\left(x^2+y^2\right)=2\left(9x^2+y^2\right),\ \ 9x^2=7y^2,\ \ x=\frac{\sqrt{7}}{3}y$	A1	Correct ratio for x & y (any equivalent form)
	direction deflected by $\tan^{-1}\frac{y}{x}+\tan^{-1}\frac{y}{3x}$	M1 A1	To find the correct angle Correct in x & y
	$=\tan^{-1}\sqrt{\frac{27}{5}}+\tan^{-1}\sqrt{\frac{3}{5}}=104.5°\quad(104)$	A1 [8]	

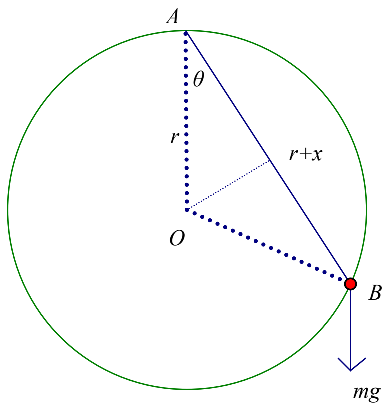
Question Number	Scheme	Marks	Notes
alt2			
	Speed perpendicular to wall after collision = $\dfrac{u\sin\theta}{3}$	B1	
	Speed parallel to the wall is unchanged	B1	
	$\dfrac{u^2}{4}=\dfrac{u^2}{9}\sin^2\theta+u^2\cos^2\theta$	M1 A1	Use the speeds to form an equation in u & θ (or equivalent) Correct unsimplified
	$27\cos^2\theta=5\sin^2\theta,\ \ \tan^2\theta=\dfrac{27}{5}$	A1	Correct trig ratio for θ (or equivalent)
	deflected by $\theta+\alpha$, $\tan(\theta+\alpha)=\dfrac{\tan\theta+\frac{1}{3}\tan\theta}{1-\frac{1}{3}\tan^2\theta}\ \ \left(=-\sqrt{15}\right)$	M1 A1	To find the correct angle Correct in θ (or equivalent)
	$\theta+\alpha=104.5°\ \ (104)$	A1 [8]	

Question Number	Scheme	Marks	Notes
4a			
	Relative velocity triangle	M1	Seen or implied
	$\frac{\sin 145}{12}=\frac{\sin\theta}{6}$, $\theta = 16.7°$	M1	Use of trig to find a relevant angle
	Bearing = $15+(180-145-16.7)=33.3°$	M1	To find the required angle
	Bearing 033°	A1 [4]	They were asked for an answer "to the nearest degree". Accept N 33º E
4b	$\frac{{}_Av_B}{\sin 18.3}=\frac{12}{\sin 145}$	M1	Correct method to find the relative velocity
	${}_Av_B = 6.58(\text{km h}^{-1})$	A1	
	Time taken = $\frac{20}{6.58}$ (hrs)	M1	For their 6.58
	Time is 3:02 pm (1502)	A1 [4]	

Question Number	Scheme	Marks	Notes
5a			
	CLM: $mx+3my=3m\times u\cos\beta-m\times 3u\cos\alpha=mu\quad(x+3y=u)$	M1 A1	Terms of correct structure but condone sign errors
	NEL: $x-y=\frac{1}{5}(3u\cos\alpha+u\cos\beta)\left(=\frac{1}{5}\left(u+\frac{2}{3}u\right)=\frac{1}{3}u\right)$	M1 A1	equation of correct structure but condone sign errors
	$x=\frac{u}{2}$, or $y=\frac{u}{6}$	DM1 A1	Dependent on the two previous M marks. Solve for x or y Correct for their x or y
	Magnitude of the impulse on A = $mu-\left(m\times-\frac{u}{2}\right)=\frac{3mu}{2}$	M1 A1 [8]	Must be positive

Question Number	Scheme	Marks	Notes
5b	Component of velocity perpendicular to the line of centres before = component after = $3u\sin\alpha = 3u\times\frac{\sqrt{8}}{3} = \sqrt{8}u$	B1	
	KE lost = $\frac{m}{2}\left(9u^2 - \left(8u^2 + \frac{1}{4}u^2\right)\right)\left[= \frac{3}{8}mu^2\right]$	M1 A1	Change in KE. Does not need to be a fraction at this stage. Does not need to include the (cancelling) component perpendicular to the line of centre. Correct unsimplified
	Fraction lost = $\frac{3/8}{9/2} = \frac{3}{8}\times\frac{2}{9} = \frac{1}{12}$	A1 [4]	

Question Number	Scheme	Marks	Notes
6a	$m\ddot{x} = 4mv - \frac{5ma \times x}{a}$ $\quad v = -\dot{x}$	M1 M1	Equation of motion as far as $m\ddot{x} = \pm 4mv - T$ Use of $v = -\dot{x}$
	$\ddot{x} + 4\dot{x} + 5x = 0$ **	A1 [3]	Reach **given answer** correctly.
6b	AE $m^2 + 4m + 5 = 0$, $m = \frac{-4 \pm \sqrt{4^2 - 4\times 5}}{2} = -2 \pm \mathrm{i}$	M1	Solve AE to find GS
	$x = \mathrm{e}^{-2t}\left(A\cos t + B\sin t\right)$	A1	
	$t = 0, x = a = A$	M1 A1	Use $t = 0, x = a$ to find A
	$\dot{x} = -2\mathrm{e}^{-2t}\left(a\cos t + B\sin t\right) + \mathrm{e}^{-2t}\left(-a\sin t + B\cos t\right)$	M1	Differentiate and use boundary conditions to find B
	$t = 0, \quad \dot{x} = 0 = -2a + B \quad x = \mathrm{e}^{-2t}\left(a\cos t + 2a\sin t\right)$	A1 [6]	
6c	String goes slack when $x = \mathrm{e}^{-2t}\left(a\cos t + 2a\sin t\right) = 0$		
	$\cos t = -2\sin t, \quad \tan t = -\frac{1}{2}$	M1 A1	Set $x = 0$ and solve for t or $\tan t$
	$\dot{x} = -2\mathrm{e}^{-2t}\left(a\cos t + 2a\sin t\right) + \mathrm{e}^{-2t}\left(-a\sin t + 2a\cos t\right)$	M1	Substitute a positive value of t to find the speed. An answer of 0.88... indicates a negative t.
	$= \mathrm{e}^{-2t}\left(-5a\sin t\right) = -0.01\ldots a \quad$ Speed $= 0.011a$ (ms^{-1})	A1 [4]	The question specifies 2 sf

Question Number	Scheme	Marks	Notes
7a			
	Measuring GPE from A, GPE $=-mg\cos\theta(r+x)$	B1	Or $-2mgr\cos^2\theta$, or $-mgr(1+\cos 2\theta)$ or equivalent
	EPE $=\dfrac{kmgx^2}{2r}$	B1	
	From the isosceles triangle, $\cos\theta=\dfrac{x+r}{2r}$	B1	
	$V=-mg\cos\theta(r+x)+\dfrac{kmgx^2}{2r}$	M1	Correct unsimplified total
	$=-mg\cos\theta\times 2r\cos\theta+\dfrac{kmgr^2\left(2\cos\theta-1\right)^2}{2r}$	A1	In terms of r & θ
	$=mgr\left\{-2\cos^2\theta+2k\cos^2\theta-2k\cos\theta+\dfrac{k}{2}\right\}$		
	$=2mgr\left\{\left(k-1\right)\cos^2\theta-k\cos\theta\right\}+\text{constant}$ **	A1 (6)	Reach **given answer** correctly

Question Number	Scheme	Marks	Notes
7b	$V = 2mgr\left(2\cos^2\theta - 3\cos\theta\right)$+constant		
	$V' = 2mgr\left(-4\cos\theta\sin\theta + 3\sin\theta\right)$	M1 A1	Differentiate V
	$V' = 0 \Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{3}{4}$	M1	Derivative = 0 and solve for θ
	$\theta = 0$ or $\theta = \pm 0.72$ rads	A3	-1 for each missing solution
	$V'' = 2mgr\left(-4\cos 2\theta + 3\cos\theta\right)$	M1	Second derivative of V
	$\theta = 0, V'' = -2mgr < 0$, unstable equilibrium	A1	Need to see $-2mgr$ or equivalent
	$\cos\theta = \frac{3}{4}$, $V'' = \frac{7mgr}{2} > 0$, stable equilibrium	A1 (9)	Do not need to consider the symmetrical position as well